Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a})$ $\quad\quad$ $\int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da}$ $\quad\quad$ $\int (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$

Cartesian Coordinates: $\overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ $\quad\quad$ $d\tau = dx\, dy\, dz$

$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$ $\quad\quad$ $\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$ $\quad\quad$ $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical Coordinates: $x = r\, sin\theta\, cos\phi,\ y = r\, sin\theta\, sin\phi,\ z = r\, cos\theta$

$\overrightarrow{dl} = dr\hat{r} + r\, d\theta\hat{\theta} + r\, sin\theta\, d\phi\hat{\phi}$ $\quad\quad$ $d\tau = r^2 sin\theta\, dr\, d\theta\, d\phi$

$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$ $\quad\quad$ $\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\, sin\theta}\frac{\partial(sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\, sin\theta}\frac{\partial A_\phi}{\partial \phi}$

$\nabla \times \vec{A} = \frac{1}{r\, sin\theta}\left(\frac{\partial(sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(r\, A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\phi}$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$

Cylindrical Coordinates: $x = s\, cos\phi,\ y = s\, sin\phi,\ z = z$

$\overrightarrow{dl} = ds\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z}$ $\quad\quad$ $d\tau = s\, ds\, d\phi\, dz$

$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$ $\quad\quad$ $\nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial(s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

$\nabla \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$ $\quad\quad$ $\nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

Dirac Delta Function: $\delta^3(\overrightarrow{\Delta r}) = \frac{1}{4\pi}\nabla \cdot \left(\frac{\widehat{\Delta r}}{\Delta r^2}\right),\ \int \delta^3(\overrightarrow{\Delta r})\, d\tau = 1$ if $\vec{r}'$ contained in volume

Irrotational Functions: $\nabla \times \vec{A} = 0$ $\quad\quad$ $\vec{A} = \nabla f$ $\quad\quad$ $\oint \vec{A} \cdot \overrightarrow{dl} = 0$

Solenoidal Functions: $\nabla \cdot \vec{F} = 0$ $\quad\quad$ $\vec{F} = \nabla \times \vec{A}$ $\quad\quad$ $\oint \vec{F} \cdot \overrightarrow{da} = 0$

Electric Field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\,d\tau'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\,da'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\,dl'$

Gauss's Law: $\oint \vec{E}\cdot\vec{da} = Q_{enc}/\varepsilon_0$, $\nabla\cdot\vec{E} = \rho/\varepsilon_0$, $\oint \vec{D}\cdot\vec{da} = Q_{f\_enc}$, $\nabla\cdot\vec{D} = \rho_{free}$

Electric Potential: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r\prime})}{\Delta r}d\tau'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r\prime})}{\Delta r}da'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r\prime})}{\Delta r}dl'$
$\vec{E} = -\nabla V$, $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot\vec{dl}$, $\nabla^2 V = -\rho/\varepsilon_0$

Laplace's Equation: $\nabla^2 V = 0\ if\ \rho = 0$

Separation of Variables: $\frac{d^2X}{dx^2} = C_1X, \frac{d^2Y}{dy^2} = C_2Y, \frac{d^2Z}{dz^2} = C_3Z$, $C_1+C_2+C_3=0$, $V = X(x)Y(y)Z(z)$

Separation of Variables (Spherical): $V(r,\theta) = \sum_0^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(cos\theta)$

Multipole Expansion: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_0^\infty \frac{1}{r^{n+1}}\int (r')^n P_n(cos\alpha)\rho(\vec{r}')d\tau'$

Dipoles: $\vec{p} = \int \vec{r}'\rho(\vec{r}')d\tau'$, $V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$, $\vec{\tau} = \vec{p}\times\vec{E}$, $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$, $\vec{P} = \vec{p}/volume$

Boundary Conditions: $\Delta V = 0$, $\Delta\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} = -\Delta\left(\frac{\partial V}{\partial n}\right)\hat{n}$, $\Delta D_\perp = \sigma_f$, $\Delta\vec{D}_{||} = \Delta\vec{P}_{||}$

Work and Energy: $W = Q\Delta V$, $W_{electrostatic} = \frac{\varepsilon_0}{2}\int E^2 d\tau$, $W_{elec+polarization} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,d\tau$

Dielectrics: $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$, $\sigma_b = \vec{P}\cdot\hat{n}$, $\rho_b = -\nabla\cdot\vec{P}$

Linear Dielectrics: $\vec{P} = \varepsilon_0\chi_e\vec{E}$, $\vec{D} = \varepsilon\vec{E} = \varepsilon_r\varepsilon_0\vec{E} = (1+\chi_e)\varepsilon_0\vec{E}$, $\rho_b = -\frac{\chi_e}{1+\chi_e}\rho_f$

Lorentz Force: $\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$, On Wire: $\vec{F}_{mag} = \int I(\vec{dl}\times\vec{B})$

Continuity: $\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

Magnetic Field of a Steady Current: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r\prime})\times\widehat{\Delta r}}{\Delta r^2}d\tau'$, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r\prime})\times\widehat{\Delta r}}{\Delta r^2}da'$
$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r\prime})\times\widehat{\Delta r}}{\Delta r^2}dl' = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl\prime}\times\widehat{\Delta r}}{\Delta r^2}$